Problem 2.17

The two equations (2.36) give a projectile's position (x, y) as a function of t. Eliminate t to give y as a function of x. Verify Equation (2.37).

Solution

Equation (2.36) is on page 54 and gives the x- and y-coordinates for a projectile moving in a medium with linear air resistance.

$$\begin{cases} x(t) = v_{xo}\tau(1 - e^{-t/\tau}) \\ \\ y(t) = (v_{yo} + v_{\text{ter}})\tau(1 - e^{-t/\tau}) - v_{\text{ter}}t \end{cases} \tag{2.36}$$

Solve the first equation for t,

$$x = v_{xo}\tau(1 - e^{-t/\tau})$$

$$\frac{x}{v_{xo}\tau} = 1 - e^{-t/\tau}$$

$$e^{-t/\tau} = 1 - \frac{x}{v_{xo}\tau}$$

$$\ln e^{-t/\tau} = \ln\left(1 - \frac{x}{v_{xo}\tau}\right)$$

$$-\frac{t}{\tau} = \ln\left(1 - \frac{x}{v_{xo}\tau}\right)$$

$$t = -\tau \ln\left(1 - \frac{x}{v_{xo}\tau}\right),$$

and substitute it into the second equation.

$$y = (v_{yo} + v_{\text{ter}})\tau\left(\frac{x}{v_{xo}\tau}\right) - v_{\text{ter}}\left[-\tau\ln\left(1 - \frac{x}{v_{xo}\tau}\right)\right]$$

$$y = \frac{v_{yo} + v_{\text{ter}}}{v_{xo}}x + v_{\text{ter}}\tau\ln\left(1 - \frac{x}{v_{xo}\tau}\right) \tag{2.37}$$

This is Equation (2.37) on page 54.